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NOTES

ON

ARITHMETIC

BY

R. K. VINTER, M.A.,

St. John's College, Cambridge,

Second Master of Queen Mary's School, Walsall.

LONDON:

SIMPKIN, MARSHALL & CO.

WALSALL: T. KIRBY.

1877.

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138.

PRICE SIXPENCE.

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T. KIRBY, PRINTER, WALSALL.

THESE Notes were, in the first instance, put together for the use of pupils preparing for Public Examinations, to aid them in calling to remembrance the various processes employed in the solution of Arithmetical Questions.

The Compiler has been induced to print them, feeling confident that they will not only be helpful in refreshing the memory of those about to compete in any examination, but will also supply an *actual want* in those Schools where *Examples only* are used.

THE MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

ALIQUOT PARTS

of a Penny.

1 farthing = $\frac{1}{4}$
2 farthings = $\frac{1}{2}$.

of a Shilling.

d.		s.
1	=	$\frac{1}{12}$
1½	=	$\frac{1}{8}$
2	=	$\frac{1}{6}$
3	=	$\frac{1}{4}$
4	=	$\frac{1}{3}$
6	=	$\frac{1}{2}$

of a £.

s.	d.		£
0	6	=	$\frac{1}{40}$
0	8	=	$\frac{1}{30}$
1	0	=	$\frac{1}{20}$
1	3	=	$\frac{1}{16}$
1	4	=	$\frac{1}{15}$
1	8	=	$\frac{1}{12}$
2	0	=	$\frac{1}{10}$
2	6	=	$\frac{1}{8}$
3	4	=	$\frac{1}{6}$

of a £

s.	d.		£
4	0	=	$\frac{1}{5}$
5	0	=	$\frac{1}{4}$
6	8	=	$\frac{1}{3}$
10	0	=	$\frac{1}{2}$

of a Cwt.

lbs.		cwt.
7	=	$\frac{1}{16}$
8	=	$\frac{1}{14}$
14 or 1 stone	=	$\frac{1}{8}$
16	=	$\frac{1}{7}$
28 or 1 qr.	=	$\frac{1}{4}$
56 or 2 qrs.	=	$\frac{1}{2}$

of a Ton.

cwt.		ton.
1	=	$\frac{1}{20}$
2	=	$\frac{1}{10}$
2½	=	$\frac{1}{8}$
4	=	$\frac{1}{5}$
5	=	$\frac{1}{4}$
10	=	$\frac{1}{2}$

of a Quarter.

lb.		qr.
1	=	$\frac{1}{28}$
1¾	=	$\frac{1}{16}$
2	=	$\frac{1}{14}$
3½	=	$\frac{1}{8}$
4	=	$\frac{1}{7}$
7	=	$\frac{1}{4}$
14	=	$\frac{1}{2}$

of a lb.

oz.		lb.
1	=	$\frac{1}{16}$
2	=	$\frac{1}{8}$
4	=	$\frac{1}{4}$
8	=	$\frac{1}{2}$

of an Acre.

		acre.
16 poles	=	$\frac{1}{10}$
20 „	=	$\frac{1}{8}$
1 rood	=	$\frac{1}{4}$
2 „	=	$\frac{1}{2}$

AVOIRDUPOIS.

16 drams = 1 ounce
16 ounces = 1 pound
14 pounds = 1 stone
28 pounds = 1 quarter
4 qrs., or 112 lbs. = 1 cwt.
20 cwt. = 1 ton

This weight is used for all goods, and for metals, *except* gold and silver.

TROY.

For gold and jewels, &c.

24 grains = 1 pennyweight
20 dwts. = 1 ounce
12 ounces = 1 pound

See *Note* 42 for the connection between Troy and Avoirdupois weights.

APOTHECARIES'.

20 grains = 1 scruple ℈
3 scruples = 1 dram ʒ
8 drams = 1 ounce ℥
12 ounces = 1 pound *lb.*

The grain, ounce, and pound are the same as in Troy Weight.

LENGTH.

4 inches = 1 hand
9 inches = 1 span
12 inches = 1 foot
3 feet = 1 yard
6 feet = 1 fathom
5½ yards = 1 rod, pole, or perch
40 poles (220 yds.) = 1 furlong
8 furlongs (1760 yds.) = 1 mile
3 miles = 1 league

SURFACE.

144 sq. inches = 1 sq. foot
9 sq. feet = 1 sq. yard
30¼ sq. yards = 1 sq. rod, pl. or pr.
40 perches = 1 rood
4 roods = 1 acre
4840 sq. yards = 1 acre
640 acres = 1 sq. mile
10 sq. chains = 1 acre

The chain for measuring land is 22 yards long, and is divided into 100 links.

CAPACITY.

2 pints = 1 quart
4 quarts = 1 gallon
2 gallons = 1 peck
4 pecks = 1 bushel
8 bushels = 1 quarter
5 quarters = 1 load

A barrel of beer = 36 galls.
A hhd. of beer = 54 galls.
A kild. of beer = 18 galls.
A butt of beer = 108 galls.
A hhd. of wine = 63 galls.
A pipe of wine = 2 hhds.

CLOTH.

2¼ inches = 1 nail
4 nails = 1 quarter
4 quarters = 1 yard
5 quarters = 1 English ell

CUBIC MEASURE.

1728 cub. inches = 1 cub. foot
27 cub. feet = 1 cub. yard
40 ft. of rough, and 50 ft. of hewn timber = 1 ton or load

TIME.

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
52 weeks & 1¼ day, or 365¼ days = 1 year
100 years = 1 century
A lunar month = 28 days

Thirty days hath September,
April, June, and November:
February has twenty-eight alone,
And all the rest have thirty-one:
But leap year coming once in four,
February then has one day more.

PAPER.

24 sheets = 1 quire
20 quires = 1 ream
10 reams = 1 bale

APOTHECARIES' FLUID MEASURE.

60 minims =1 fluid dram
8 fluid drams =1 fluid ounce
20 fluid ounces =1 pint
8 pints =1 gallon

N.B.—A pint of water weighs a pound and a quarter.

GEOMETRY.

60 seconds (″) =1 minute (′)
60 minutes =1 degree (°)
90 degrees =1 quadrant
360 degrees, or 4 quadrants = 1 circle

A right angle contains 90 degrees.

QUARTER DAYS.

Lady-day, 25th of March. Midsummer, 24th of June. Michaelmas, 29th of September. Christmas, 25th of December.

THE METRIC SYSTEM.

LENGTH.

The unit of length is the Mètre, and is the ten-millionth part of a line, supposed to be drawn along the meridian, from the pole to the equator.

10,000 metres = 1 myriametre
1,000 metres = 1 kilometre
100 metres = 1 hectometre
10 metres = 1 dekametre
1 metre
·1 metre = 1 decimetre
·01 metre = 1 centimetre
·001 metre = 1 millemetre

N.B.—11 metres = 12 yards nearly, and the metre = 39·37 inches nearly.

SURFACE.

The square metre is the unit of surface. In land, a square dekametre is taken as the unit, and is called the "are."

				Sq. Metres
Hectare	=	100 ares	=	10,000
Dekare	=	10 ares	=	1,000
Are	=	1 are	=	100
Centare	=	$\frac{1}{100}$ are	=	1

The "are" is $\frac{1}{40}$th part of an acre *nearly*.

CAPACITY.

The cubic decimetre called the "litre" is taken as the unit.

				Cub. Metre
Kilolitre	=	1000 litres	=	1
Hectolitre	=	100 litres		
Dekalitre	=	10 litres		
Litre	=	1 litre		
Decilitre	=	$\frac{1}{10}$ litre		
Centilitre	=	$\frac{1}{100}$ litre		

A litre is rather more than 1¾ pints; therefore 50 litres = 11 gallons nearly.

WEIGHT.

The unit is the "gramme" which is the weight of a cubic centimetre of distilled water.

Millier = 1,000,000 grammes
Quintal = 100,000 grammes
Myriagram = 10,000 grammes
Kilogram = 1,000 grammes
Hectogram = 100 grammes
Dekogram = 10 grammes
Gram = 1 gramme

The gram = 15⅖ grains Troy nearly.

The unit of money is the *franc* which weighs 5 grammes: 100 centimes = 1 franc.

NOTES ON ARITHMETIC.

1. **Unit** or **Unity** is the name given to that quantity which is to be reckoned *one* when other quantities of the same kind are mentioned.

2. **Number** is the relation of a quantity to its unit. An **Abstract Number** has reference to no particular kind.

3. A **Concrete Number** has reference to a particular kind, as 1 yard, 1 lb., &c.

4. **Notation** is the art of expressing numbers by figures or symbols.

5. Abstract numbers can be multiplied by abstract numbers; concrete numbers can be multiplied by abstract numbers; but concrete numbers cannot be multiplied by concrete numbers. (*For an exception, see Note* 15.)

In Division, if the dividend be concrete, the divisor may be abstract or concrete.

6. To explain the process in Subtraction:

Ex. 9263 To subtract 5 from 3 is impossible; so separate
7495 1 ten from 6 tens, and adding it to the 3 units,
—— say 5 from 13 leaves 8. Now we are supposed to
1768 have separated 1 ten from 6 tens, but as the figure 6 really remains the same, we still have to take 1 from it; also, we have to take 9 from it; so instead of taking away first 1 and then 9 more, take away 10 at once; but 10 from 6 being impossible, separate 1 from the place of hundreds, and adding it as 10 tens to the 6 tens, say 10 from 16 leaves 6, and so on.

7. The process of Multiplication is a short method of repeated addition:

$$\textit{Ex.}\quad 752 \times 257 = \left\{\begin{array}{lcr} 752 \times 200 & = & 150400 \\ 752 \times 50 & = & 37600 \\ 752 \times 7 & = & 5264 \end{array}\right.$$

$$\begin{array}{r} \hline 193264 \end{array}$$

8. The process of Division is a short method of repeated subtraction, and is, therefore, the exact converse of multiplication.

9. One number is a **measure** of another when it measures, that is divides that other an exact number of times.

10. The **Greatest Common Measure** of several numbers is the greatest number which divides each of the several numbers an exact number of times.

11. One number is a **multiple** of another when it contains, that is can be divided by that other an exact number of times.

12. A **Common Multiple** of several numbers is a number which contains each of the several numbers an exact number of times. The **least common multiple** is the *least* number which will contain the numbers.

13. **Factors** of a number are those which, being multiplied together, produce the number.

A prime factor is a number which is divisible only by itself and unity. The best method of resolution into factors is to divide the number by 2, 3, 5, 7, 11, &c., in succession.

The following Rules for finding factors should be remembered:

(1) A number is divisible by 2 when it ends in 0 or an even integer.

(2) A number is divisible by 3 when the sum of the digits is.

(3) A number is divisible by 4 when the number consisting of its last two digits is: *e.g.*, 720. The number 20 is divisible by 4, therefore 720 is.

(4) A number is divisible by 5 when it ends in 0 or 5.

(5) A number is divisible by 6 when it is divisible by 2 and 3: *i.e.*, it must end in an even number, and the sum of its digits be divisible by 3.

(6) A number is divisible by 8 when the number consisting of its last three digits is: *e.g.*, 7440. The number 440 is divisible by 8, therefore 7440 is.

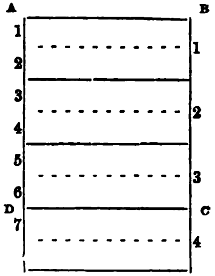
(7) A number is divisible by 9 when the sum of the digits is.

(8) A number is divisible by 11 when the difference of the sum of the digits in the odd and even places is.

14. To prove that a fraction is not altered by multiplying numerator and denominator by the same number:

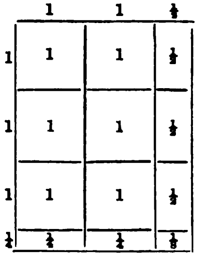
Ex. $\frac{3}{4} = \frac{6}{8}$

A B C D contains 3 quarters and also 6 eighths.



15. Illustration of $3\frac{1}{4}$ft. × $2\frac{1}{2}$ft.:

The area in the figure is composed of 6 squares, each containing 1 sq. ft.; 3 rectangles, each containing $\frac{1}{2}$ sq. ft.; 2 rectangles, each containing $\frac{1}{4}$ sq. ft.; and 1 rectangle containing $\frac{1}{8}$ sq. ft.



∴ The whole area:

$$1+1+1+1+1+1+\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{4}+\tfrac{1}{4}+\tfrac{1}{8}=8\tfrac{1}{8} \text{ sq. ft.}$$

16. To *Carpet a Room.* Multiply the length by the breadth (this gives the area of the floor); divide the product by the width of the carpet (this gives the length of carpet required).

Ex. A room 18ft. by 12ft. is to be covered with carpet $\frac{2}{3}$ yd. wide: find the expense of doing it at 10s. per yard.

By the rule $(\frac{18}{3}\times\frac{12}{3})\div\frac{2}{3}\times 10 =$ expense $=$ 360*s.* $=$ £18.

17. To *Paper a Room.* To obtain the area of the side wall, multiply the length by the height; for the area of the end wall, multiply the breadth by the height; by doubling each of these, we get the area of the two side and two end walls, and adding these two results together we obtain the area of the four walls. This process is obviously equivalent to adding the length and breadth together and multiplying by twice the height.

$$2\ H\ (L+B)$$

Ex. Paper a room whose length is 25ft., breadth 15ft., and height 20ft., with paper 2ft. wide at 4½d. per yard.

Area of walls $= 2 \times 20\ (25+15) = 1600$ sq. ft., with paper 2ft. wide, quantity required $= \frac{1600}{2} = 800$ lineal ft. and at 4½d. per *yard* $= \frac{800}{3} \times \frac{9}{2} = 1200d. = £5.$

18. A **Fraction** is a quantity which represents a part or parts of an integer or whole. It consists of *numerator* and *denominator.*

19. The **Denominator** shews into how many parts the whole is divided.

20. The **Numerator** shews how many of such parts are taken.

21. A **Proper Fraction** is one whose numerator is less than the denominator.

22. An **Improper Fraction** is one whose numerator is equal to or greater than the denominator.

23. A **Compound Fraction** is a fraction of a fraction, as $\frac{2}{3}$ of $\frac{3}{4}$.

24. A **Complex Fraction** is one in which either the numerator or denominator or both are fractions, as $\frac{2\frac{1}{3}}{3}$, $\frac{3\frac{1}{2}}{4\frac{1}{4}}$

25. A **Mixed Number** is a whole number and a fraction, as $5\frac{3}{7} = 5 + \frac{3}{7}$.

26. To reduce a fraction to its lowest terms: divide numerator and denominator by their greatest common measure.

27. To bring a mixed number to an improper fraction: multiply the whole number by the denominator and add in the numerator: this will give the new numerator, and the denominator is the same as before.

28. To reduce an improper fraction to a mixed number: divide the numerator by the denominator; the quotient gives the *whole number*, and the remainder (if any) as numerator with the denominator, forms the fraction.

29. To reduce fractions to the least common denominator: find the L. C. M. of the denominators; this is the new denominator; divide *each* of the denominators into this denominator and multiply its numerator by the quotient.

ADDITION OF FRACTIONS.

30. Reduce the fractions to the L. C. D.; add together the numerators thus formed and place the denominator below this sum: if there result an improper fraction reduce it to a mixed number.

SUBTRACTION OF FRACTIONS.

31. Reduce the fractions to their L. C. D., and subtract the numerators thus formed, and place under this difference the common denominator.

MULTIPLICATION OF FRACTIONS.

32. Strike out all factors that are common to both numerator and denominator; multiply all the remaining numerators together for a new numerator, and all the remaining denominators together for a new denominator.

See Note 15.

DIVISION OF VULGAR FRACTIONS.

33. Invert the divisor, and proceed as in multiplication.

Ex. 1. Divide $14\frac{2}{3}$ by $8\frac{4}{5}$

i.e. $\frac{44}{3} \div \frac{44}{5}$

i.e. $\frac{44}{3} \times \frac{5}{44} = \frac{5}{3}$

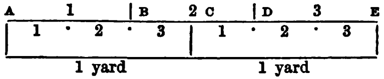
Ex. 2. Divide $\frac{2}{3}+\frac{5}{6}$ of $\frac{9}{10}+\frac{7}{12}$ by $\frac{5}{9}+\frac{4}{7}$ of $\frac{21}{40}+\frac{13}{90}$

$$\frac{2}{3}+\frac{5}{6} \text{ of } \frac{9}{10}+\frac{7}{12}=\frac{2}{3}+\frac{3}{4}+\frac{7}{12}=\frac{8+9+7}{12}=\frac{24}{12}=2$$

$$\frac{5}{9}+\frac{4}{7} \text{ of } \frac{21}{40}+\frac{13}{90}=\frac{5}{9}+\frac{3}{10}+\frac{13}{90}=\frac{50+27+13}{90}=\frac{90}{90}=1$$

$\therefore$ The Ans. $=2+1=2$

34. To shew that $\frac{2}{3}$ of $1 = \frac{1}{3}$ of 2:



Here, A B which is $\frac{2}{3}$ of 1 yard $= \frac{1}{3}$ of A E $= \frac{1}{3}$ of 2 yards.

35. In adding mixed fractions together take the whole numbers and then the proper fractions.

36. *Caution:* $\frac{1}{2} + \frac{1}{3}$ of $\frac{3}{7} + \frac{5}{6}$ does not mean $\left(\frac{1}{2} + \frac{1}{3}\right)$ of $\left(\frac{3}{7} + \frac{5}{6}\right)$
but $\frac{1}{2} + \left(\frac{1}{3} \text{ of } \frac{3}{7}\right) + \frac{5}{6}$

37. Reduce 8*s*.$6\frac{6}{7}$*d*. to the fraction of £1:

$$6\tfrac{6}{7}d. = \tfrac{48}{7} \times \tfrac{1}{12}s. = \tfrac{4}{7}s.$$
$$8\tfrac{4}{7}s. = £\tfrac{60}{7} \times \tfrac{1}{20} = £\tfrac{3}{7}$$

38. Find the value of £$\frac{5}{9}$:

$$\tfrac{5}{9} \times 20s. = \tfrac{100}{9} = 11\tfrac{1}{9}s. = 11 + \tfrac{1}{9}s.$$
$$\tfrac{1}{9}s. = \tfrac{12}{9} = 1\tfrac{1}{3}d.$$
$$\therefore £\tfrac{5}{9} = 11s. 1\tfrac{1}{3}d.$$

39. *Notice.*—If $\frac{5}{8}$ths of a ship be worth £20; then $\frac{1}{8} =$ £4
∴ the ship $= \frac{8}{8} =$ £32.

40. A can do a piece of work in 4 days which B can do in 3 days; how long will they take to complete the work?

A can do $\frac{1}{4}$th in *one* day
B „ $\frac{1}{3}$rd „
∴ A and B together can do $\frac{1}{4} + \frac{1}{3}$ in *one* day
∴ „ „ „ $\frac{7}{12}$ths „
∴ they will finish the work in $\frac{12}{7}$ or $1\frac{5}{7}$ days.

41. **Continued Fractions:** $1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4}}}$

$3 + \frac{1}{4} = \frac{13}{4}$ ∴ fraction $= 1 + \cfrac{1}{2 + \cfrac{1}{\frac{13}{4}}} = 1 + \cfrac{1}{2 + \frac{4}{13}}$

$2 + \frac{4}{13} = \frac{30}{13}$ ∴ fraction $= 1 + \cfrac{1}{\frac{30}{13}} = 1 + \frac{13}{30} = 1\frac{13}{30}$

42. 7,000 grains Troy = 1lb. Avoirdupois
5,760 „ „ = 1lb. Troy

$$\therefore \frac{\text{1lb. Troy}}{\text{1lb. Avoird.}} = \frac{5760}{7000} = \frac{144}{175}$$

43. Reduce 18*s*.8*d*. to the fraction of half-a-crown:

$18s.8d. = 18\frac{2}{3}s. = \frac{56}{3}s.$ and half-a-crown $= \frac{5}{2}s.$

$\therefore$ fraction required $= \frac{56}{3}s. \div \frac{5}{2}s. = \frac{56}{3} \times \frac{2}{5} = \frac{112}{15}$

44. **Decimals** are fractions which always have 10 or some power of 10 for their denominator:

Ex. 576·576 = 5 hundreds + 7 tens + 6 units + 5 tenths + 7 hundredths + 6 thousandths.

45. In *Subtraction* or *Addition* of decimals place the decimal points under each other and perform the operations indicated as with ordinary numbers.

46. In *Multiplication* multiply as if ordinary numbers and point off in the product as many places (beginning from the right hand) as there are in the multiplier and multiplicand together.

DIVISION OF DECIMALS.

47. Remove the decimal place of the *divisor* to the end (*i.e.* to the right), thus making the divisor a whole number. Shift the point in the *dividend* a corresponding number of places (towards the right). Proceed as in ordinary division, and put down the point in the quotient as soon as arrived at in the course of division:

Examples.

```
(1)  ·16) ·8 (       (2)  ·16) 800· (       (3)  16000) ·0080000 (
     i.e.                 i.e.
      16) 80· ( 5·         16) 80000· ( 5000
          80                   80
          —                    —
                                000
                               ———
     Ans.: 5              Ans.: 5000             Ans.: ·0000005
```

48. Reducing a vulgar fraction to a decimal is simply division of decimals, and therefore follows the rule above.

49. To reduce a circulating decimal to a vulgar fraction

$$\begin{aligned} \text{Let}\quad x &= {\cdot}5151\ \&c. \\ 100x &= 51{\cdot}5151\ \&c. \\ 99x &= 51 \\ \therefore\ x &= \tfrac{51}{99} = \tfrac{17}{33} \end{aligned}$$

Hence the rule for a *pure* circulating decimal.

50. Mixed circulating decimal:

$$\begin{aligned} \text{Let}\quad x &= {\cdot}779\,\dot{3}\dot{4}\ \&c. && (1) \\ 1000x &= 779{\cdot}3434\ \&c. && (2) \\ 100000x &= 77934{\cdot}\dot{3}\dot{4}\ \&c. && (3) \end{aligned}$$

Subtracting (2) from (3)

$$\begin{aligned} 99000x &= 77934-779 \\ \therefore\ x &= \frac{77934-779}{99000} = \frac{77155}{99000} = \&c. \end{aligned}$$

Hence the rule—Subtract the non-recurring figures from the circulating decimal written as a whole number; this gives the numerator: for the denominator place as many 9's as there are recurring figures, followed by as many ciphers as there are non-recurring figures.

51. To reduce any quantity or fraction of one denomination to the decimal of another denomination:

Ex. (1) 17*s.* 1½*d.* to decimal of £1:

$$\begin{array}{r|l} 2 & 1{\cdot} \\ \cline{2-2} 12 & 1\ {\cdot}5\ \textit{pence} \\ \cline{2-2} 20 & 17{\cdot}125\ \textit{shillings} \\ \cline{2-2} & \quad {\cdot}85625\ \textit{pounds} \end{array}$$

Ex. (2) 2*fur.* 50*yds.* 2*ft.* 3*in.* to decimal of a furlong:

$$\begin{array}{rr|l} & 12 & 3{\cdot} \\ \cline{3-3} & 3 & 2\ {\cdot}25\ \textit{feet} \\ \cline{3-3} 220\Big\{ & 20 & 50{\cdot}75\ \textit{yards} \\ \cline{3-3} & 11 & \ 2{\cdot}5375 \\ \cline{3-3} & & 2{\cdot}2306\dot{8}\dot{1}\ \textit{furlongs} \end{array}$$

52. Converse proposition.

Ex. (1) Find the value of ·81375 of a £1:

$$\begin{array}{r l} \pounds 81375 & \\ 20 & \\ \hline 16{\cdot}27500 & \textit{shillings} \\ 12 & \\ \hline 3{\cdot}300 & \textit{pence} \end{array}$$

Ans.: 16*s.* $3\frac{3}{10}$*d.*

Ex. (2) Find the value of ·3725 of a ton:

$$\begin{array}{r l} {\cdot}3725 & \textit{ton} \\ 20 & \\ \hline 7{\cdot}4500 & \textit{cwt.} \\ 4 & \\ \hline 1{\cdot}80 & \textit{qrs.} \end{array}$$

Ans.: 7*cwt.* $1\frac{4}{5}$*qrs.*

REASON FOR THE RECURRENCE OF THE FIGURES OF THE QUOTIENT IN THE SAME ORDER.

53. "In attempting to reduce a vulgar fraction to a decimal the *remainder* at each step of the division must always be *less* than the divisor, that is, the denominator of the vulgar fraction; and the number of remainders different from each other which can arise, can only be a number less than the units of the divisor. If, therefore, the remainder never become zero, by carrying on the division far enough, one remainder must occur again, and as a cipher is added to every remainder, when the dividend becomes the same as has occurred before, the quotient will necessarily be again the same, and the process from that point will be repeated.

"Hence the *greatest* number of figures which it is possible for the period of a circulating decimal to contain is *one less* than the number of units in the denominator of the fraction from which it springs."—*Girdlestone.*

PROPORTION.

54. The *ratio* of one quantity to another *means* the number of times the first contains or is contained in the second.

A *ratio* is written thus, as 6 : 2 or $\frac{6}{2}$ *i.e.* 6 to 2.

(The first term is the *antecedent*, the second is the *consequent*.)

By the above is meant, that 6 is three times greater than 2.

55. *Proportion* is the equality of ratios; thus

6 : 2 is the same as 9 : 3

$\therefore \frac{6}{2}=\frac{9}{3}$ or as it may be written

6 : 2 :: 9 : 3

6 and 3 are the *extremes*, 2 and 9 are the *means*

Notice that $6 \times 3 = 2 \times 9$

and generally if $a : b :: c : d$ then $a\,d = b\,c$ (1)

i.e., the product of the extremes = product of the means.

$$\text{From (1)}\quad a=\frac{b\,c}{d}\quad b=\frac{a\,d}{c}\quad c=\frac{a\,d}{b}\quad d=\frac{b\,c}{a}\qquad (2)$$

Thus, if *any* three terms of a proportion be given the fourth can be found from (2).

56. *Ex.* (1) If 15 sheep cost £45, what number of sheep may be bought for £63?

15 sheep : Ans. (sheep) :: £45 : £63

∴ from the second formula of (2)

$$Ans. = \frac{15 \times 63}{45} = 21 \text{ sheep.}$$

57. *Ex.* (2) A bankrupt's $\left\{\begin{matrix}\text{liabilities}\\ \text{debts}\end{matrix}\right\}$ are £5000, and his $\left\{\begin{matrix}\text{assets}\\ \text{effects}\end{matrix}\right\}$ are £1250; what dividend (*i.e.*, how much in the £) can he pay?

total debts : a debt of :: money he has *Ans.* :

£5000 : 20*s.* :: £1250 : (No. of shillings in £ he pays)

$$\therefore\ Ans. = \frac{1250 \times 20}{5000} = 5s.$$

58. Notice the method of working the following example:

If 9 men in 4 days consume 12 loaves; how many loaves will 6 men consume in 6 days?

9 men in 4 days consume 12 loaves

1 man in 4 days consumes $\frac{12}{9}$ loaves

1 man in 1 day consumes $\frac{12}{9\times4}$ loaves

6 men in 1 day consume $\frac{12\times6}{9\times4}$ loaves

6 men in 6 days consume $\frac{12\times6\times6}{9\times4}$ loaves

Ans.: 12 loaves.

PROPORTIONAL PARTS.

59. *Ex.* (1) Divide 64 into parts in the ratio of 1 : 3. This is equivalent to dividing 64 into 4 parts, and taking one part for one number and the remaining three parts for the other number.

The statement, therefore, is

$$4 : 1 :: 64 : \textit{Ans.} \therefore \textit{Ans.} = 16$$
$$4 : 3 :: 64 : \textit{Ans.} \therefore \textit{Ans.} = 48$$

and generally to divide a given number into parts in the ratio of 1 : 2 : 3 : 4, &c.

We have for the first part

$$1+2+3+4+\&c. : \textit{given number} :: 1 : \textit{Ans.} = \text{1st part}.$$

60. **Interest** is a sum of money paid for the use of money.

When the Interest of the Principal (*i.e.*, the given sum) alone is taken it is called Simple: but if the Interest, as soon as due, is added to the Principal, and interest charged on the whole *i.e.*, on Principal + Interest, it is called *Compound* Interest.

61. **Rate** per cent. per annum means the interest on £100 for one year.

The working of examples in Interest (Simple) depends on a simple rule of Proportion.

Ex. Find the Simple Interest on £250 for 4 years at 5 per cent. per annum.

The Interest on £100 for 4 years at 5 per cent. = £20.

$$\therefore \text{£}100 : \text{£}250 :: \overset{\text{Interest}}{\text{£}20} : \overset{\text{Interest}}{\textit{Ans.}} = \text{£}50$$

62. **Compound Interest**=a series of Simple Interest examples.

The most convenient and general way of working is as follows:

Ex. The Compound Interest on £450 10*s.* for 3 years at 4 per cent.

$$\begin{array}{rcl}
450.5 & - & \textit{1st Principal} \\
4 & & \\
\hline
18.020 & & \textit{1st year's Interest} \\
450.5 & & \\
\hline
468.52 & = & \textit{2nd Principal} \\
4 & & \\
\hline
18.7408 & = & \textit{2nd year's Interest} \\
468.52 & & \\
\hline
487.2608 & = & \textit{3rd Principal} \\
4 & & \\
\hline
19.490432 & = & \textit{3rd year's Interest} \\
18.7408 & = & \textit{2nd year's Interest} \\
18.02 & = & \textit{1st year's Interest} \\
\hline
56.251232 & = & \textit{Compound Interest for 3 years}
\end{array}$$

Ans.= £56 5*s.* 0·29568*d.*

QUESTIONS ON SIMPLE INTEREST.

63. (1) Required the *time* in which a certain sum will produce a given *interest* at a given *rate*.

$$\left.\begin{array}{l}\text{Interest on given sum}\\ \text{for 1 year at the rate}\end{array}\right\} : \left.\begin{array}{c}\text{Total interest}\\ \text{(given)}\end{array}\right\} :: \begin{array}{c}\text{Year}\\ 1\end{array} : \begin{array}{c}\text{Time}\\ \textit{Ans.}\end{array}$$

Ex. In what time will £20 produce £20 interest at 5 per cent. per annum?

$$\begin{array}{c}\text{Int. on given sum for 1 yr.}\\ \dfrac{20\times5\times1}{100}\end{array} : \begin{array}{c}\text{Total Interest}\\ \pounds20\end{array} :: \begin{array}{c}\text{Year}\\ 1\end{array} : \begin{array}{c}\text{Years}\\ \textit{Ans.}=20\end{array}$$

64. (2) Required the *rate* at which a certain sum will produce a certain interest in a given time.

$$\left.\begin{array}{c}\text{Interest on given sum for}\\ \text{the time at 1 per cent.}\end{array}\right\} : \left.\begin{array}{c}\text{Total}\\ \text{interest}\end{array}\right\} :: \begin{array}{c}\text{Per cent.}\\ 1\end{array} : \begin{array}{c}\textit{Ans.}\\ \text{rate per cent.}\end{array}$$

Ex. At what rate will £20 become £40 (or produce £20 interest) in 10 years?

$$\begin{array}{c}\text{Int. on given sum at 1 per c.}\\ \dfrac{\pounds20\times10\times1}{100}\end{array} : \begin{array}{c}\text{Total Interest}\\ \pounds20\end{array} :: \begin{array}{c}\text{Per cent}\\ 1\end{array} : \begin{array}{c}\textit{Ans. rate p. c.}\\ \text{10 p. c.}\end{array}$$

65. (3) Required the *principal* which will produce a certain interest (or amount to a certain sum) in a given *time* at a given *rate.*

$$\left.\begin{array}{c}\text{Int. } \textit{or} \text{ amt. on}\\ \pounds100 \text{ for the}\\ \text{time at the rate}\end{array}\right\} : \left.\begin{array}{c}\text{given interest}\\ \textit{or}\\ \text{given amount}\end{array}\right\} :: \overset{\text{Principal}}{\pounds100} : \overset{\textit{Ans:}}{\text{Reqd. Principal}}$$

Ex. What Principal will amount to £40 in 10 years at 10 per cent. per annum ?

$$100+\left(\frac{100\times10\times10}{100}\right) : \overset{\text{Amount}}{\pounds40} : \pounds100 : \overset{\textit{Ans.}\text{ Principal}}{\pounds20}$$

66. When a debt is paid *before* due a sum smaller than the actual debt may be paid down by the debtor and will be accepted by the creditor as payment in full. The money thrown off is *Discount*, and the money paid is the *Present Worth* of the debt.

$\therefore$ Debt = Present Worth + Discount.

If £100 were due a year hence, and if £95 were accepted as the present worth of the debt, the £95 being put out to interest at 5 per cent. would *not* gain £5 and would *not* amount to £100 at the end of the year: and £5 would be too large a sum to allow as the discount on £100 for a year. But if £105 were due a year hence and £100 were accepted as Present Worth, the £100 being put out to interest at 5 per cent. *would* gain £5 and *would* amount to £105 at the end of the year.

Hence we see that £5, the *interest* on £100 for 1 year, is the *discount* on £105 for the same period: and generally the same sum that is the interest of £100 for any time, will, for the same time, be the discount on £100 increased by that interest.

Hence we have the following statements for examples:

$$\left.\begin{array}{c}\pounds100 + \text{ its interest for}\\ \text{given time at given rate}\end{array}\right\} : \pounds100 :: \text{given sum} : \text{present worth}$$

$$\left.\begin{array}{c}\pounds100 + \text{ its interest for}\\ \text{given time at given rate}\end{array}\right\} : \text{its interest} :: \text{given sum} : \text{discount}$$

STOCKS.

67. Distinguish between *money in the funds* and *ready cash.*

For example:—When Stocks are selling *at* say £94, this means that £100 *stock* is selling for £94 *cash.*

When a person *invests* so much money in the funds, this means that he takes so much *cash* and buys as much *stock* as he can at the quotation. On the other hand, *selling out* is selling *stock* for as much *cash* as it will produce at the market price of the day.

The *Income* obtained from *stock* is easily calculated by Proportion:

Ex. The Income arising out of £5000 stock in the 3½ per cent.

Stock		Stock		Income		
£100	:	£5000	::	£3½	:	Income required.

The *Income* derived from *investing* so much cash in any stock will depend on the price of stock at the time of purchase. Hence, the cheaper the stock, the greater in proportion will be the Income produced.

When £100 is selling at £100 cash, the stock is said to be at *par.*

All questions in Stocks are worked by Proportion.

Ex. (1) What quantity of stock can be bought for £3600 when at 90?

Cash		Cash		Stock		Stock
£90	:	£3600	::	£100	:	*Ans.* = £4000.

Ex. (2) What money will be obtained by selling out £7000 stock at 92?

Stock		Stock		Cash		Cash
£100	:	£7000	::	£92	:	*Ans.* = £6440.

Ex. (3) What income will be obtained from £38400 invested in 4 per cent. at £96?

Cash		Cash		Income		Income
£96	:	£38400	::	4	:	*Ans.* = £1600.

Ex. (4) What income will be obtained from £12000 stock in the 3 per cent. at 92?

Stock		Stock		Income		Income
£100	:	£12000	::	3	:	*Ans.* = £360.

Ex. (5) What will be the difference of income by the transfer of £1600 stock in the 4 per cents. at 90 to the 3 per cents. at 72?

$$\overset{\text{Stock}}{\pounds 100} : \overset{\text{Stock}}{\pounds 1600} :: \overset{\text{Income}}{4} : \text{1st income} \;\therefore\; \text{1st income} = \pounds 64$$

$$\text{also, } \overset{\text{Stock}}{\pounds 100} : \overset{\text{Stock}}{\pounds 1600} :: \overset{\text{Cash}}{\pounds 90} : \text{cash obtained by selling out}$$

$$\textit{Ans.} = \pounds 1440$$

This money is to be invested in the 3 per cents. at 72:

$$\therefore \overset{\text{Cash}}{\pounds 72} : \overset{\text{Cash}}{\pounds 1440} :: \overset{\text{Income}}{\pounds 3} : \text{2nd income}$$

$$\text{2nd income} = \pounds 60$$

$$\therefore \text{Difference in income} = \pounds 4.$$

PROFIT AND LOSS.

68. Beware of the error that *gain per cent* means gain on a *hundred articles*: it is the gain which £100 laid out upon such articles would bring in.

The things necessary to be considered are—

(1) Prime cost or cost price
(2) Retail price, *i.e.*, selling price
(3) The gain or loss per cent. which will be incurred by the transaction.

Hence all questions will have one or other of these statements.

$$\text{cost price} : \text{retail price} :: \pounds 100 : \pounds 100 + \text{gain per cent.}$$

$$\text{or, cost price} : \text{retail price} :: \pounds 100 : \pounds 100 - \text{loss per cent.}$$

Ex. (1) An article cost 5*s.* and was sold for 6*s.*; what is the gain per cent.?

$$\overset{\text{Cost price}}{5s.} : \overset{\text{Selling price}}{6s.} :: \pounds 100 : \pounds 100 + \text{gain per cent.}$$

$$\therefore \pounds 100 + \text{gain per cent.} = \frac{6 \times 100}{5} = \pounds 120$$

$$\therefore \text{gain per cent.} = 20.$$

Ex. (2) An article which cost 5*s.* was sold at a loss of 20 per cent.; what was its selling price?

$$\overset{\text{Cost price}}{5s.} : \textit{Ans.} :: \pounds 100 : \overset{\pounds 80}{\pounds 100 - 20}$$

$$\textit{Ans.} = \frac{5 \times 80}{100} = 4s.$$

PARTNERSHIP.

69. A field is rented by two persons for £35; the former keeps in it 5 oxen for 10 weeks; the latter, 10 oxen for 2 weeks; find the rent each paid.

If a person keeps 5 oxen in a field for 10 weeks, it is equivalent to keeping 50 oxen for 1 week, and similarly the other person will have an equivalent to the keep of 20 oxen for the *same* time: hence their payments should be in the ratio of 50 : 20.

$$\therefore 50 + 20 : 50 :: \text{£}35 : \text{1st man's share} = \text{£}25.$$

A, B, C are partners in a business. A puts in £3000, B £2500, and C £2000. After 3 months, A withdraws £1000, and B £500. At the end of 3 months more, A withdraws another £1000, B and C each add £1000 to theirs. At the end of 3 months more, A puts in £2000. The profits are £590; how must they be divided?

A is represented by

$$3000 \times \overset{\text{M.}}{3} + 2000 \times \overset{\text{M.}}{3} + 1000 \times \overset{\text{M.}}{3} + 3000 \times \overset{\text{M.}}{3} = 27000$$

B is represented by

$$2500 \times \overset{\text{M.}}{3} + 2000 \times \overset{\text{M.}}{3} + 3000 \times \overset{\text{M.}}{6} = 31500$$

C is represented by

$$2000 \times \overset{\text{M.}}{6} + 3000 \times \overset{\text{M.}}{6} = 30000$$

$$\therefore A : B : C :: 270 : 315 : 300$$
$$\text{or,} :: 18 : 21 : 20$$

$$\therefore 18 + 21 + 20 : 18 :: \text{£}590 : \text{A's share} = \text{£}180$$
$$18 + 21 + 20 : 21 :: \text{£}590 : \text{B's share} = \text{£}210$$
$$18 + 21 + 20 : 20 :: \text{£}590 : \text{C's share} = \text{£}200$$

CHAIN RULE.

70. *Ex.* 7 apples are worth as much as 5 pears; 3 pears as much as 8 plums; 12 plums as much as 45 nuts; 100 nuts as much as 14 oranges; how many oranges is an apple worth?

7 apples = 5 pears
3 pears = 8 plums
12 plums = 45 nuts
100 nuts = 14 oranges
x oranges = 1 apple

$$\therefore x \times 100 \times 12 \times 3 \times 7 = 1 \times 14 \times 45 \times 8 \times 5$$

$$x = \frac{1 \times 14 \times 45 \times 8 \times 5}{100 \times 12 \times 3 \times 7} = 1$$

SQUARE ROOT.

71. The arithmetical process of extracting the square root is based upon the algebraical.

We know that the square of $a+b$ is $a^2+2ab+b^2$. Now $a+b$ is deduced from this expression in the following way:

$$\begin{array}{r|l} & a^2+2ab+b^2\ (a+b \\ & a^2 \\ \hline 2a+b & 2ab+b^2 \\ & 2ab+b^2 \\ \hline \end{array}$$

Arrange in descending powers of some letter. The square root of the first term is a : place the square of a under the first term and subtract—the remainder is $2ab+b^2$. Double a and divide $2ab$ by it—the quotient is b. Add b to $2a$ and place it as a divisor on the left of the remainder. Multiply $2a+b$ by b and place the product under $2ab+b^2$—add b to the quotient.

Thus, $a+b$ has been obtained from $a^2+2ab+b^2$.

Hence the rule for extracting the square root of a whole number.

Begin at the unit's figure and mark off the figures in pairs—find the largest number whose square can be subtracted from the first period (which may consist of *one* or *two* figures)—write this as the first figure of the root, subtract its square from the first period, and to the remainder bring down the next pair of figures. Double the first figure of the root, and take this as a trial divisor into the figures of the new dividend—place the trial quotient to the figure (or figures) already forming the divisor, and multiply the number so formed by the trial quotient, placing the product under the dividend, and subtract. To the remainder bring down the next pair of figures—double the quotient, which now consists of two figures, and proceed as before.

Ex.

$$\begin{array}{r|l} & \overparen{46}\overparen{79}\overparen{92}\overparen{81}\ (6841 \\ 6^2 & 36 \\ \hline 128 & 1079 \\ & 1024 \\ \hline 1364 & 5592 \\ & 5456 \\ \hline 13681 & 13681 \\ & 13681 \\ \hline \end{array}$$

Should the number consist of a whole number and a decimal, mark off in pairs, *right* and *left*, from the decimal point. Should the number be a decimal only, begin from the *decimal point* and mark off in pairs.

72. *Note* 58 is the common sense way of working out examples in Compound Proportion—other methods have been adopted, one of which will be sufficient to append.

Taking the same example "If 9 men, &c."

Read the question, first making the number of days *unity*, thus: "If 9 men in *one* day consume 12 loaves; how many loaves will 6 men consume in *one* day?"

$$\text{The statement is}\quad \overset{\text{Men}}{9} : \overset{\text{Men}}{6} :: \overset{\text{Loaves}}{12}$$

So making the number of men *unity* it would read "If *one* man in 4 days consume 12 loaves; how many loaves will *one* man consume in 6 days?"

$$\text{The statement is}\quad \overset{\text{Days}}{4} : \overset{\text{Days}}{6} :: \overset{\text{Loaves}}{12}$$

$$\text{The combined statement is}\quad \begin{array}{ccc} \text{Men} & & \text{Men} \\ 9 & : & 6 \\ \text{Days} & & \text{Days} \\ 4 & : & 6 \end{array} :: \overset{\text{Loaves}}{12}$$

$$\therefore\ \textit{Ans.} = \frac{6\times6\times12}{9\times4} = 12 \text{ loaves.}$$

PRACTICE.

73. *Ex.* Find the value of 3*lbs.* 4*oz.* 9*dwts.* 13½*grs.* of gold at £3 17*s.* 6*d.* an ounce.

Here 3*lbs.* 4*oz.* = 40 ounces:

		£	s.	d.	
5*dwts.*	¼*oz.*	3	17	6	*per oz.*
				10	
		38	15	0	
				4	
		155	0	0	
4*dwts.*	⅕*oz.*		19	4½	
12*grs.*	⅛ of 4*dwts.*		15	6	
			1	11¼	
1½*grs.*	⅛ of 12*grs.*			$2\frac{2}{3}\frac{9}{2}$	
		£156	17	$0\frac{2}{3}\frac{1}{2}$	

If the example had been :—find the value of 341*lbs.* 4*oz.* 9*dwt.* 13½*grs.* at £3 17*s.* 6*d.* per ounce, we must first reduce 341*lbs.* 4*oz.* to ounces = 4096*oz.* and find, by simple practice, the cost of 4096*oz.* at the price named : thus

		£
10*s.*	½ of £	4096
		3
		12288
5*s.*	½ of 10*s.*	2048
		1024
2*s.*6*d.*	½ of 5*s.*	512
		15872

		£	*s.*	*d.*
5*dwts.*	¼*oz.*	3	17	6
			4096	
4*dwts.*	⅕*oz.*	15872	0	0
			19	4½
12*grs.*	⅛ of 4*dwts.*		15	6
			1	11¼
1½*grs.*	⅛ of 12*grs.*			$2\frac{29}{32}$
		15873	17	$0\frac{21}{32}$

CUBE ROOT.

74. The arithmetical process is based upon the algebraical.

Begin at the unit's figure and mark off the figures in *trios.* Should the number, whose cube root is required, consist of a whole number and a decimal, mark off in trios *right* and *left* from the decimal point. Should the number be a decimal only, begin from the *decimal point* and mark off in *trios.*

The following example will show the process:

		12812904 | 234
	2^3 =	8
30 × 2 = 60		
3	300 × 2^2 = 1200	4812
	3 × 63 = 189	
63		
	1389	4167
2 × 3 = 6	3^2 = 9	645904
690	158700	
4	4 × 694 = 2776	
694	161476	645904

690 is 30 times the quotient already obtained : 158700 is 300 times the square of the quotient. Instead of *squaring,* the numbers 189, 1389, and 9, have been *added* together.